Lesson 006
Axioms, Interpretations, and Properties of Probability

Friday, September 22

Weather

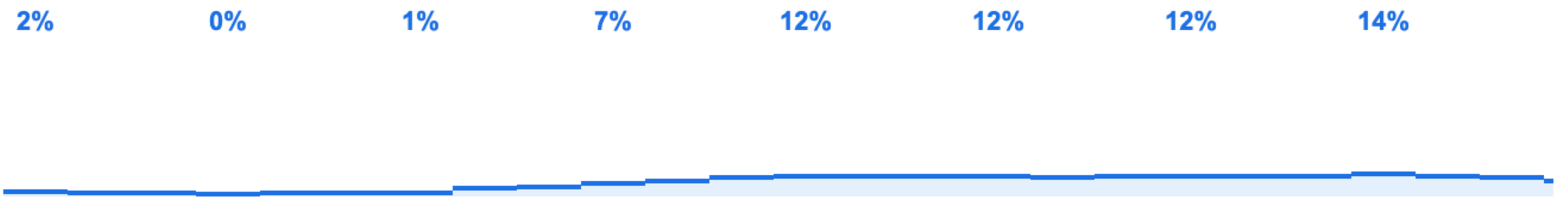
Friday

Partly cloudy

17 °C | °F

Precipitation: 10%
Humidity: 70%
Wind: 14 km/h

Temperature | Precipitation | Wind



Fri	Sat	Sun	Mon	Tue	Wed	Thu	Fri
17° 8°	17° 9°	19° 10°	18° 8°	18° 8°	19° 9°	20° 11°	19° 11°

Probability refers to a numeric value representing how likely a particular event is.

Frequentist Probability

- The **numeric value** is the long-run proportion of times that an event happens.
- What happens if we repeat an experiment over and over and over (...) and over again?

	5 Flips	**10 Flips**	**50 Flips**	**100 Flips**	**500 Flips**	**1000 Flips**	**5000 Flips**	**10,000 Flips**	**50,000 Flips**	**100,000 Flips**
Heads	2	6	31	55	252	487	2454	5051	25,029	49,934
Tails	3	4	19	45	248	513	2546	4949	24,971	50,066
Proportion Heads	**0.40000**	**0.60000**	**0.62000**	**0.55000**	**0.50400**	**0.48700**	**0.49080**	**0.50510**	**0.50058**	**0.49934**

Examples on Probability Interpretation

What does the probability mean in each of the following cases?

- The probability of drawing a diamond from a deck of cards is 0.25.
- The probability of rolling a one on a six-sided die is $\frac{1}{6}$.
- The probability that a particular candidate wins the upcoming election is 0.35.
- The probability that a machined part produced is within specification is 0.999.
- The probability that an earthquake larger than anticipated hits the region is $\frac{1}{1000000}$.

A biased coin has a probability of heads of 0.75. Which of the following is correct?

Flipping the coin 4 times will result in 3 heads.

0%

Flipping the coin $4,000,000$ times will result in $3,000,000$ heads.

0%

If n is the number of flips of the coin, then, as $n \to \infty$, the number of heads will tend to $0.75n$.

0%

None of the above.

0%

A sports forecaster uses a model to predict that the probability that a particular team wins their next game is 0.95. The team loses the game. Which of the following is true?

The forecaster was incorrect.

0%

The forecaster's model was badly calibrated.

0%

Throughout a season, assuming the forecaster makes many predictions, this result will be expected.

0%

The forecaster should probably update the model.

0%

Axioms of Probability

Positivity

$$P(A) \geq 0 \text{ for every event } A$$

Think "it is not possible to have a probability less than zero."

Axioms of Probability

Unit Measure

$$P(\mathcal{S}) = 1$$

Think "the probability that *something* happens is 1."

Axioms of Probability

Additivity

For disjoint A_i : $P\left(\bigcup_i A_i\right) = \sum_i P(A_i)$

Think "the probability of mutually exclusive events adds up."

Suppose a die is rolled. A is the event that a six shows up and B is the event that a four shows up. Which of the following does not follow from the basic axioms?

$P(A \cup A^C) = 1$

0%

$P(A \cup B) = P(A) + P(B)$

0%

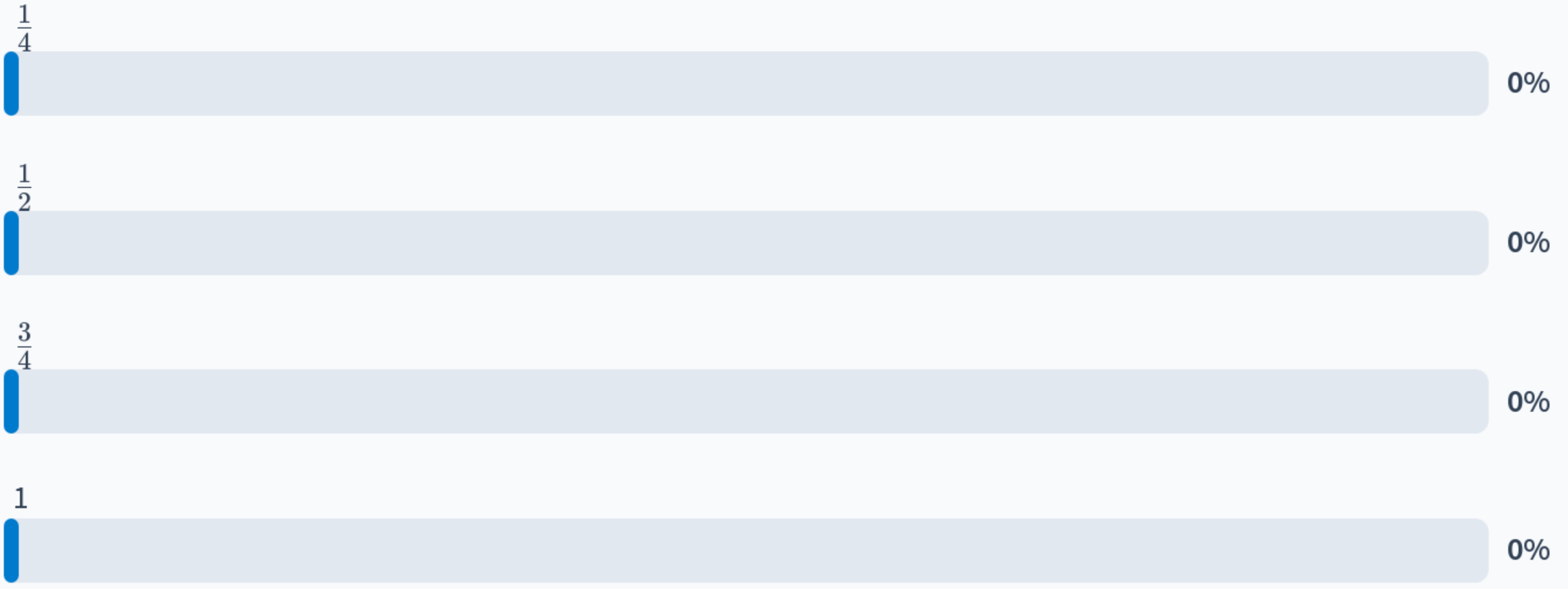
$P(A) + P(B) \geq 0$

0%

$P(A) = P(B) = \frac{1}{6}.$

0%

Suppose that $\mathcal{S} = \{1, 2, 3, 4\}$ and that the probability of each outcome is exactly $\frac{1}{4}$. What is $P(\{2, 4\})$?



Suppose that A is the event that an even number is rolled, so that $P(A) = 0.5$. Further, suppose that B is the event that a two is rolled, so that $P(B) = \frac{1}{6}$. Using the basic axioms of probability directly, what is $P(A \cup B)$?

$P(A \cup B) = \frac{4}{6}$

0%

$P(A \cup B) = \frac{1}{2}$

0%

$P(A \cup B) = \frac{1}{6}$

0%

The basic axioms of probability cannot be used to solve for this probability, directly.

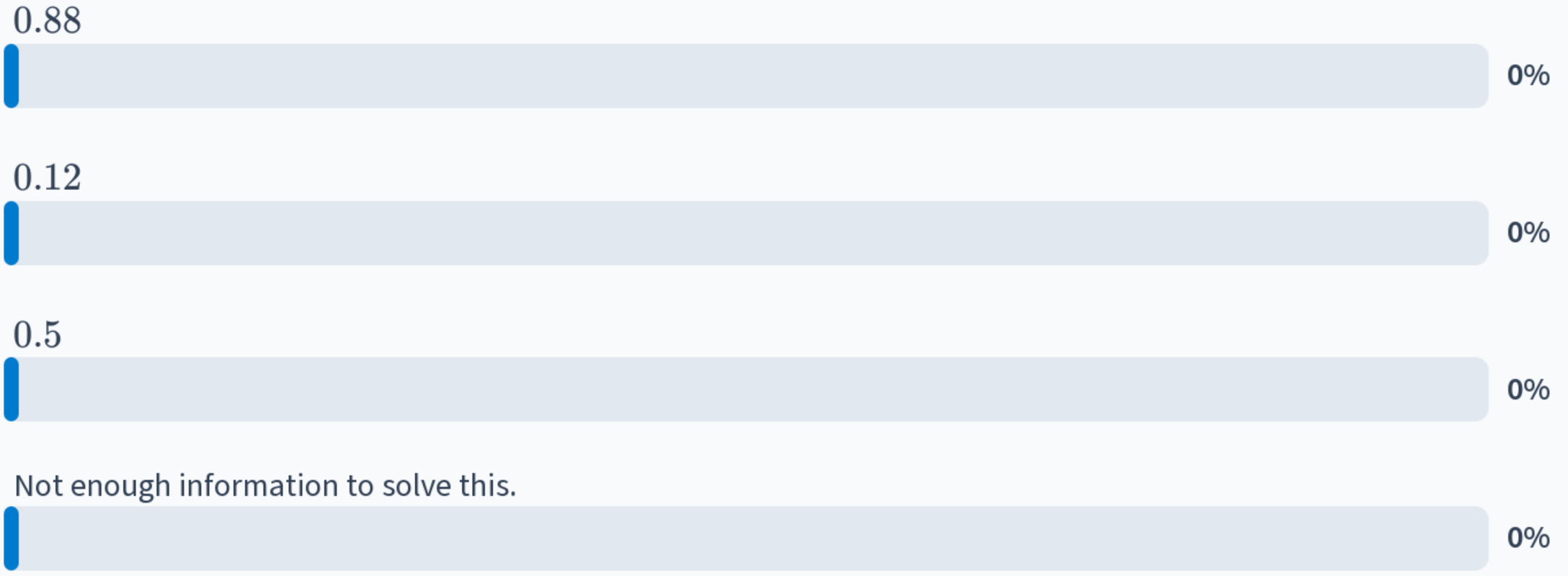
0%

Secondary Properties of Probability

Using the axioms of probability, we can show that ...

- $P(A^C) = 1 - P(A)$
- $P(\varnothing) = 0$
- $P(A) \leq 1$
- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$

The probability that a ball bearing fails during its first month is 0.12. What is the probability that it does not fail during its first month?



A ball is drawn out of a hat at random. The ball is either blue, green, or red, represented by events B, G, and R respectively. We know that $P(B) = 0.25$ and $P(G) = 0.6$. What is $P(R)$?

$P(R) = \frac{1}{3}$.

0%

$P(R) = 0.25$.

0%

$P(R) = 0.15$.

0%

$P(R) = 0.6$.

0%

A ball is drawn out of a hat at random. The ball is either blue, green, or red, represented by events B, G, and R respectively. We know that $P(B) = 0.25$ and $P(G) = 0.6$. What is $P(B \cap R)$?

$P(B \cap R) = 0.25 + 0.15 = 0.40.$

0%

$P(B \cap R) = 0.25 \cdot 0.15 = 0.0375.$

0%

$P(B \cap R) = \min\{0.15, 0.25\} = 0.15.$

0%

$P(B \cap R) = 0.$

0%

Suppose that $P(A) = 0.5$, $P(B) = 0.3$, and $P(A \cup B) = 0.75$. What is $P(A \cap B)$?

$P(A \cap B) = 0.15$ 0%

$P(A \cap B) = 0.05$ 0%

$P(A \cap B) = 0.3$ 0%

$P(A \cap B) = 0$ 0%

Your friend is very into a new cryptocurrency. They say that the probability: its price increases tomorrow is 0.40, its price doesn't change is 0.40, its price decreases is 0.30. Which statement is most correct?

If your friend is correct, buying the cryptocurrency has a 0.80 chance of not losing you money.

0%

If your friend is correct, buying the cryptocurrency has a 0.70 chance of not losing you money.

0%

Your friend is not correct.

0%

If your friend is correct, buying the cryptocurrency has a 0.40 chance of earning you money.

0%

Assigning probabilities is about **counting possibilities.**